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ABSTRACT

This investigation capitalizes upon the information processing models that depend upon measurement of latency of response to a mathematical problem and the decomposition of reaction time (RT). Simple two term addition problems were presented with possible solutions for true-false verification, and accuracy and RT to response were recorded. Total RT was hypothesized to be the sum of: (1) setting time, which involves encoding of terms and operators and initializing internal counters: (2) incrementing of retrieval time: and (3) verification time. Twelve children from each of grades two, three, and four, and 16 college undergraduates served as subjects. Students performed a true-false verification task on arithmetic problems presented on a microcomputer. Results indicate that latency to correct response is accounted for by two structural variables. Different combinations of reconstructive and reproductive processes were used by children and adults on different types of simple problems. While all adult performance was characterized by a two-stage model, the performance of young children, in general, was characterized by the reconstructive implicit counting strategy. The report states the important point is the regression models validated problem type differences corresponding to different information processing stages employed by children and adults that were also indicative of more global mathematical achievements. (MP)

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The Development from Effortful to Automatic Processing in Mathematical Cognition

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A full report of this study will be available shortly.

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The utility of information-processing models for understanding mental operations in common tasks such as reading and arithmetic has been acknowledged only recently. While reading processes have received a great deal of scientific attention, studies of arithmetic processes have been less frequent, although the acquisition and use of mathematical knowledge ranks rather close to reading in importance.

Two avenues of research in this realm have been pursued. The first employs various techniques to trace the control processes involved in simple or complex arithmetic problems. Research questions are generally aimed at conscious strategies employed by subjects in mathematical problem solving. The alternative pursuit, as in the present study, capitalizes upon the information-processing models that depend upon measurement of latency of response to a mathematical problem and the decomposition of this reaction time (RT). Simple addition is of course the building block for more complex mathematical problem solving, but even in younger children (7-10 year olds) the speed and relative ease with which subjects solve these problems often precludes a careful analysis of errors or strategies. It is for this reason that we turn to this information processing approach. In particular, simple two-term addition problems are presented with possible solutions for true-false verification, and accuracy and reaction time to response are recorded. A list of sample problems is on page 1 of your handout (see Table 1). The value of "q" is the stated sum and is either correct or differs from

correct by a certain quantity. The total reaction time is hypothesized to be the sum of: 1) setting time, which involves encoding of the terms and operators and initializing internal counters; 2) incrementing or retrieval time; and 3) verification time or comparison of stated answer to true answer. We use the verification task so that we may be able to measure across problem types differences that may give insight into the storage, retrieval, comparison, and decision processes central to human problem solving. Such a paradigm has proven useful in research on memory for linguistic material and there is no reason to expect otherwise for memory of number concepts.

In previous studies, two classes of models have been tested against the data. The first class, the "counting" models, is used to predict RT as a function of the magnitude of one of the numbers involved in the problem, e.g., the minimum or maximum addend, or even the actual sum of the problem. These models, especially RT as a function of the minumum addend, fit the data of children especially well (Groen & Parkman, 1972; Groen & Resnick, 1977), but fit some of the adult data also (Ashcraft & Battaglia, 1978; Groen & Parkman, 1972). Fits of counting models are interpreted as evidence that an implicit counting strategy is used to solve the problem, with each counting step taking some constant amount of time (for a linear fit), from about 150 msec to 500 msec depending on age of the subject.

Another class of models may be referred to as "retrieval" models, where the actual sum is retrieved from long-term memory and then compared to the stated sum. These models predict RT to be a nonlinear function of some problem parameter, e.g., SUM^2. When good fits to data

have been found, it has also been true that RT to problems with false close stated answers, +/- 2 or 3 (see handout), were longer than false far problems (Ashcraft & Battaglia, 1978). Such a finding is in line with the relationship of subjective distance to RT common to most network search models of long-term memory.

Thus, it appears that for preschool and first grade children the minimum addend strategy is used for addition. College students, however, appear to have developed strategies that utilize a process more consistent with other tasks requiring long-term memory. Between first grade and college there are as yet few data tracing this change in strategy. The purpose of the present study is to understand the development of the processes involved in computation, storage and retrieval of what certainly are to become simple, highly overlearned arithmetic concepts and solutions. Second through fourth grades have been sampled since this is a time when the school curriculum makes ever increasing demands on the use of basic arithmetical processes. College subjects were tested for appropriate comparisons. We expect that most of the developments of interest in these skills are attained by fourth grade.

Method

Subjects.

Twelve children from each of grades 2, 3, and 4 and sixteen college undergraduates served as subjects.

Materials.

All problems were presented with possible solutions of one of three types: (a) true, where the solution presented matched the actual solution of the problem; (b) false close, where the solution was different from the correct answer by plus or minus two or three; and (c) false far, where the solution was different from the correct answer by plus or minus five or six. No negative (less than zero) answers were presented, and "ties", where the digits to be added were the same, were allowed and of theoretical interest.

Procedure.

Subjects performed a true-false verification task on all arithmetic problems. Upon presentation of each problem on a microcomputer monitor, the subject responded true or false as fast as possible with a manual key press.

Results

Since results for third and fourth graders were similar, we will report results for three groups of subjects: younger children, older children, and adults.

As a first step, we attempted to isolate (for the problems as a set and for each subset), the structural variables that successfully predicted RT. We did this both for group data, averaging over subjects for each problem, and for individual subjects on all problems. Of prime interest was whether a counting model or some type of memory search and retrieval model would be aprropriate and whether the appropriate model would shift with age and problem type.

We tested a set of counting and retrieval models, assessing linear and nonlinear fits to the data. We also used results from ANOVAs in selecting models to describe the data.

The error rates and incidence of extreme RTs were uniformly low across grade (7-9%; 1.8% overall).

The first important result from the regression analyses concerns the True, Tie problems. In none of the samples did RT increase linearly with the minimum addend; yet, in all groups, there was a significant nonlinear increase in RT with increasing SUM. This would be consistent with a position asserting that such problems have been overlearned by even the youngest subjects tested, and this problem type, when presented, invokes a retrieval operation. Subjects did not use the implicit counting strategy hypothesized for other problem types and this is seen in the low B weights for the structural variable representing the minumum addend, which is interpreted as the amount of time taken for each implicit counting step.

However, with the True, Nontie problems, the slopes, at least for the children, were within the limits of implicit counting speed. Only the 13 msec slope for college subjects was too fast for implicit counting and was not significantly different from zero. Yet, the nonlinear component of the SUM^2 model was significant as well, with R^2 values in the same range as with MIN, although the values were lower for the children.

The minimum addend variable proved appropriate for False Close, Nontie problems. Slopes were greater than 100 for children but only 23 msec for adults. Yet, the retrieval model was inadequate even for these

adults. Such a result would be obtained if adults were using the retrieval strategy on most but not all trials and reverted to a counting process on a few trials.

A lack of fit when the minimum addend and SUM^2 each are used separately as structural variables, as in the False Far, Nontie problems, might be consistent with a "fast-no" strategy. When the stated and actual sums do not correspond reasonably, then no retrieval operation need be employed. Only a first stage of matching a rough estimate of the sum (magnitude estimate) need be compared with the stated sum and a rapid "false" response made. The second stage, or inequality judgment stage, would be reserved for those problems with a reasonable stated sum (i.e., the True and False Close, Nontie problems) with latencies for these problems not very different. Such an ordering of RT, in increasing order for the False Far, Nontie; True, Nontie; and False Close, Nontie problems, was found for the two samples of children. Latencies to Tie problems were shorter than for Nonties. For children, Tie problems may indeed be solved with the non-counting search and retrieval process. For college subjects, however, True problems were solved faster than False problems but Ties and NonTies appeared not to be solved by different processes. Perhaps for these subjects some two-stage model in which the actual sum is first retrieved and then compared to the stated sum is appropriate for all problems. At any rate, there was evidence of some development with the type of process employed changing from childhood to adulthood.

Individual differences.

While the individual subjects' analyses were redundant with the group analyses for the most part, it was apparent that the range of strategies used within any of the age groups was very large. Overall, the SUM^2 was more appropriate for Tie problems with minimum addend more appropriate for NonTies. But again there was some difference in appropriateness between children and adults.

Correlations were computed for each problem type and grade with RT as well as the intercept and slope values from the regression equations for the counting and retrieval models and measures from the mathematics section of the Stanford Achievement Test. For the youngest subjects, there were strong relationships between RT, particularly in the form of the minimum addend, and mathematics achievement. The faster the inferred implicit counting speed, the greater the mathematics achievement. The only correlational evidence for the role of retrieval processes was manifested by the correlation between the B weight for SUM^2 and computation skills for older children, with greater evidence for the memory retrieval process related to higher achievement.

Discussion

In summary, latency to correct response has been accounted for by two structural variables in particular: the minimum addend, which represents the value by which an internal counter is incremented to achieve a sum and SUM^2, representing the nonlinear component of an indirect memory search and retrieval operation used when reconstructive processes are no longer necessary.

From these latencies and model fits, we can conclude that different combinations of reconstructive and reproductive processes were used by children and adults on different types of simple arithmetic problems. College subjects' performance can be characterized by a two-stage process: 1) Reproductive processes function first to retrieve the correct sum or product from memory, and 2) then this retrieved number is compared with that stated in the problem. Of course, decision and response always follow. For the children, Tie problems were solved in a similar fashion.

While all adult performance was characterized by the simple two-stage model, the performance of young children (excluding the True, Tie problems) was characterized by the reconstructive implicit counting strategy. Problem solutions were reconstructed with an effortful, time-consuming yet accurate counting operation. For the most part, developmental changes occurred after grade 2, and the change thereafter was of some significance. This could be described as only quantitative since the results could be accounted for by arguing that older children already manifested adult-like reproductive processes, but only infrequently. More likely, the changes occurring beyond this time are a combination of these quantitative changes within individuals and qualitative changes across subjects, with more adopting the reproductive strategies.

The important point is that the regression models (and parameters thereof) validated problem type differences corresponding to different information-processing stages employed by children and adults that were also indicative of more global mathematical achievements.

References

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Table 1. Two-term addition problems
P + R = Q
addends stated sum

Problem Type	Example [1]	Grade	M RT(msec)	Model Fits			
				slope	R^2 [a]	slope	R^2 [b]
True (Tie)	2+2=4	2	2221	0	.00	12**	.80**
		3/4	1680	11	.07	6*	.68*
		Col	707	5	.03	3**	.75**
True (Nontie)	3+1=4	2	3502	444****	.67	29****	.68****
		3/4	2282	208****	.61	6*	.60****
		Col	819	13	.09	2**	.24**
False Close (Nontie)	2+6=10	2	3933	257***	.52	0	.65****
		3/4	2514	136***	.56	0	.60****
		Col	880	23***	.54	0	.59***
False Far (Nontie)	7+2=4	2	3350	258	.50	9	.48**
		3/4	2220	98	.21	0	.24
		Col	860	11	.13	0	.21

Notes: Models: a) RT= Minimum addend ; significance of slope and R^2 are equivalent.

b) $RT = SUM + SUM^2$; slope is value of B weight for SUM^2 (nonlinear) component. R^2 can still be high if SUM^2 is nonsignificant since SUM is correlated with minimum addend as a predictor of RT

1 These are sample problems. Close values for "Q" are ± 2 or 3; far values are ± 5 or 6 of actual sum

* $p<.05$ ** $p<.01$ *** $p<.001$ **** $p<.0001$